CSE 5526 - Autumn 2019

Introduction to Neural Networks

Homework #4

Due Tuesday, Nov. 12

Grader: Hao Zhang
Office: 474 Dreese Lab
Office hours: 2:30-3:30 T & W
Email: zhang.6720@osu.edu

Problem 1. For a winner-take-all network with 5 neurons, the function of each neuron is defined as

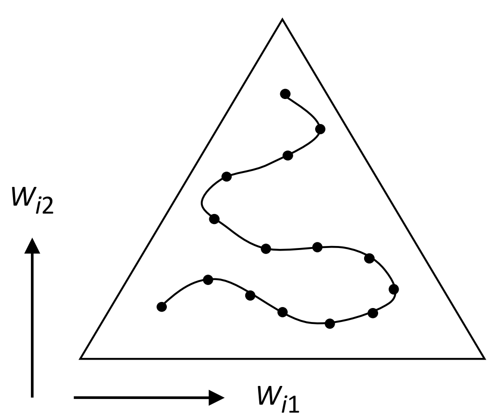
$$y_i(t+1) = \varphi\left((S-1)y_i(t) - \sum_{j \neq i} y_j(t)\right)$$

where S is the number of output neurons, and the activation function is defined as

$$\varphi(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$$

The above network receives the input vector at time step 0, $\mathbf{x}^T = (0.2, 0.2, 0.3, 0.4, 0.3)$. Find the network output at time step 1 and 2.

Problem 2. The following figure shows the final weight vectors of a self-organizing map that has been trained on two-dimensional input vectors which were drawn from a uniform distribution over the triangular area. Lines between units (represented by dots) connect neighboring neurons. Draw the diagram of a network that has undergone such self-organization. Specify the elements of the network and their connections (no detailed values are needed).



Problem 3. For a Boltzmann machine operating at temperature T. Assuming no self-connection, prove that neuron i flips from state x_i to $-x_i$ with the following probability:

$$P(x_i \to -x_i) = \frac{1}{1 + \exp(\frac{\Delta E_i}{T})}$$

where ΔE_i is the energy change resulting from such a flip. (HINT: consider the cases of $x_i = -1$ and $x_i = 1$ separately.)