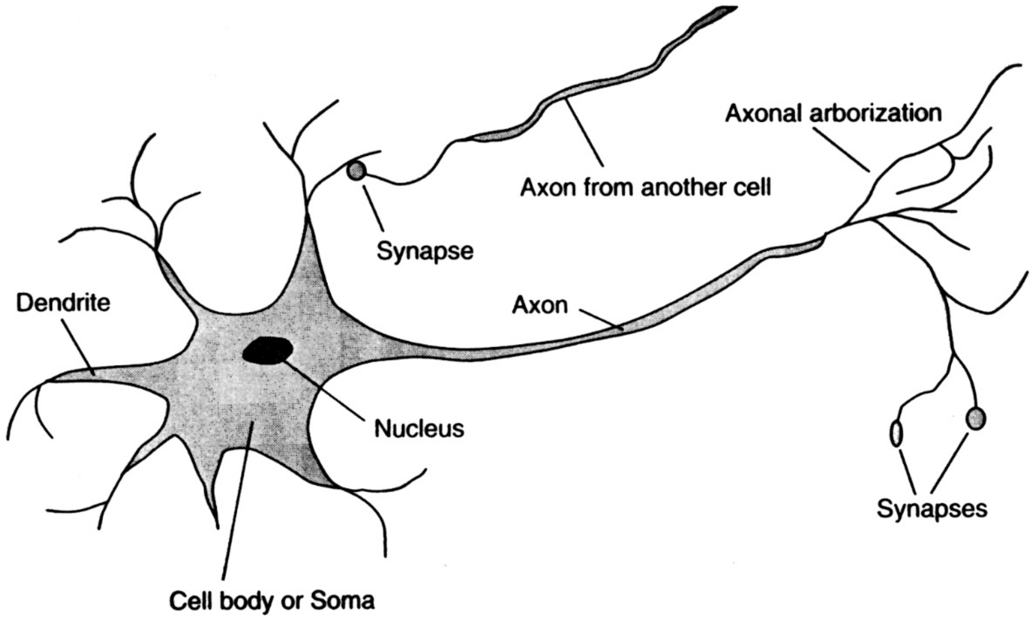
Artificial Intelligence

Intro to Neural Networks

How the Brain Works (sort of)



- Neuron is fundamental functional unit
  - Soma: cell body
  - Axon: long single fiber that connects to other neurons
  - Dendrites: connected to axons from other neurons
  - Synapse: connecting junction

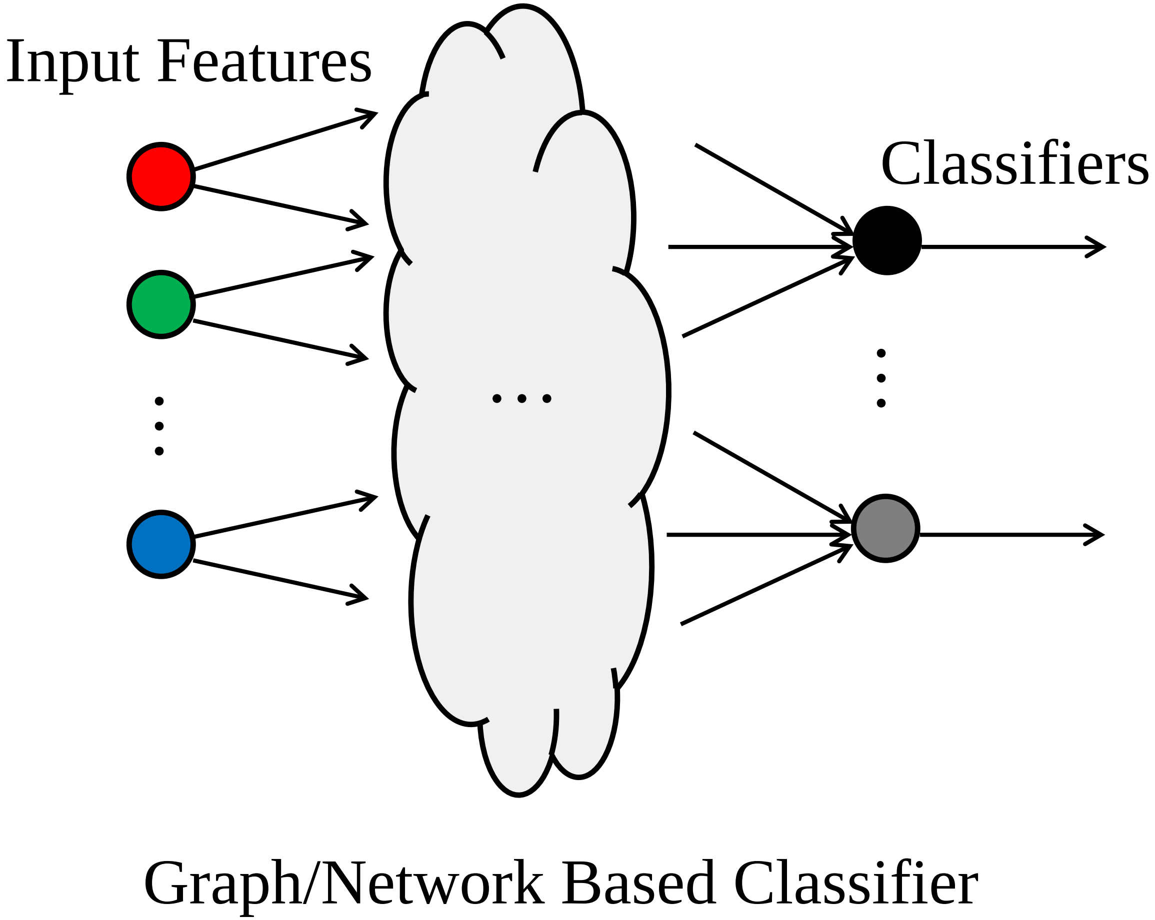
How the Brain Works

- Signals propagated between neurons by electrochemical reaction
  - Chemical substances released from synapses and enter dendrite, raising or lowering electric potential of cell body
    - Synapses that increase potential are <u>excitatory</u>
    - Synapses that decrease potential are <u>inhibitory</u>
  - <u>Action potential</u> (electrical pulse) sent down axon when electric potential of cell body reaches a threshold

How the Brain Works

- A collection of simple cells can lead to thought, action, and consciousness
  - Bottom-up statement
  - Long way from a theory of consciousness
  - “Brains cause minds” (Searle 1992)

Neural Networks



Neural Networks

- Neural net is composed of nodes (units)
  - Some connected to outside world as input or output units
- Nodes are connected by links
  - Input and output links
- Each link has numeric weight associated with it
  - Primary means of **long-term storage/memory**
  - Weights are modified to bring network's input/output behavior to goal response
- Nodes have activation level
  - Given its inputs and weights
  - Local computation based on inputs from neighbors (no global control)

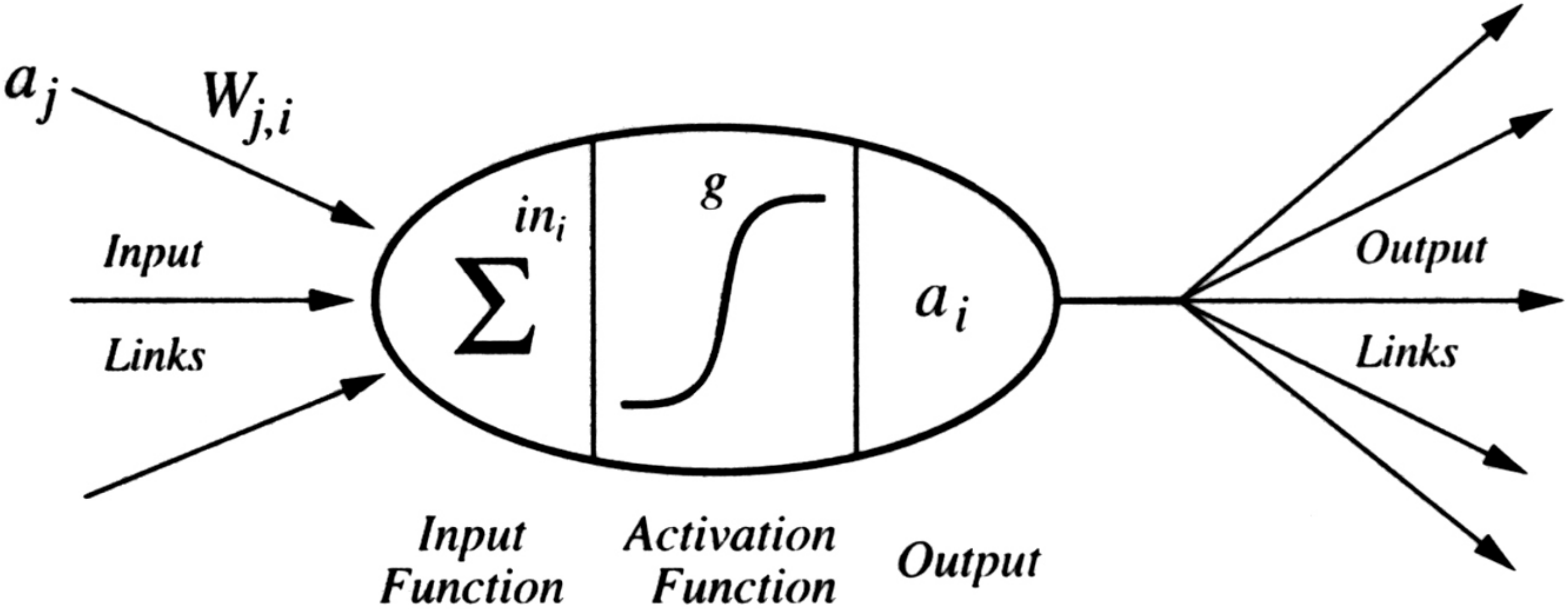
Using Neural Networks

- One must first decide
  - How many nodes to use
  - What kind of nodes are appropriate
  - How nodes are to be connected into a network
- Weights are randomly initialized, then training learns correct weight values given a particular set of training examples
  - Input examples are labeled with correct outputs
  - One must decide how to encode examples in terms of network inputs and outputs

Simple Computing Elements

- Each unit performs simple computation
  - Receives signals from input links
  - Computes new activation level
  - Sends activation level along each output link
- Computation split into two components
  - Linear input function
    - Computes weighted sum of inputs
  - Nonlinear activation function
    - Transforms weighted sum into final activation value

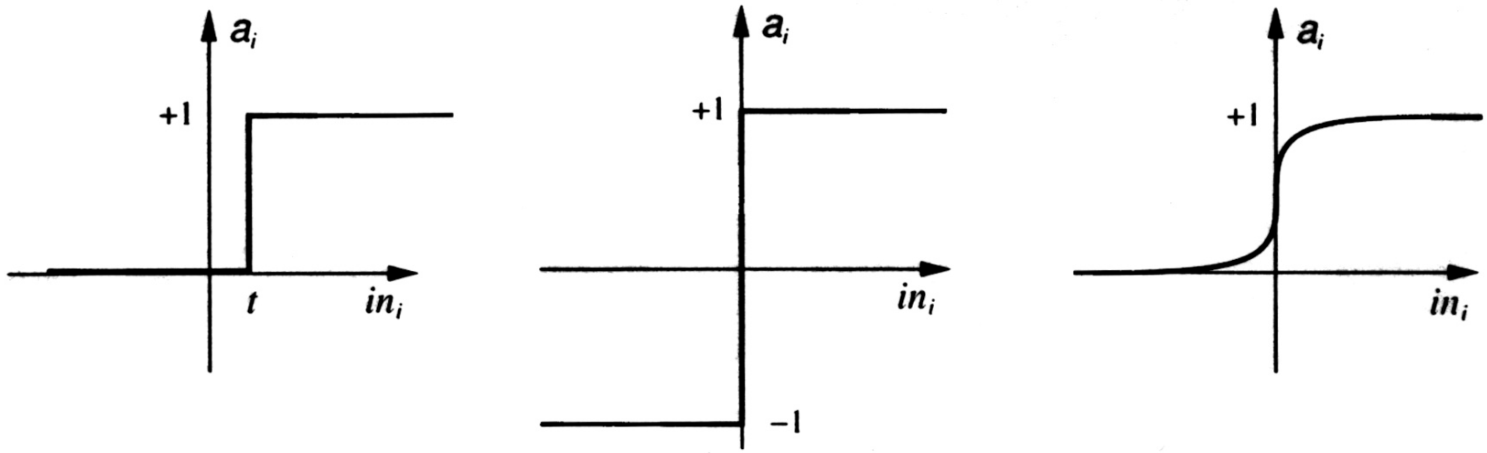
Simple Computing Elements



$$\sum_j W_{j,i} a_j = \boldsymbol{W} \cdot \boldsymbol{a}$$

$$g\left(\sum_j W_{j,i} a_j\right)$$

Types of Activation Functions



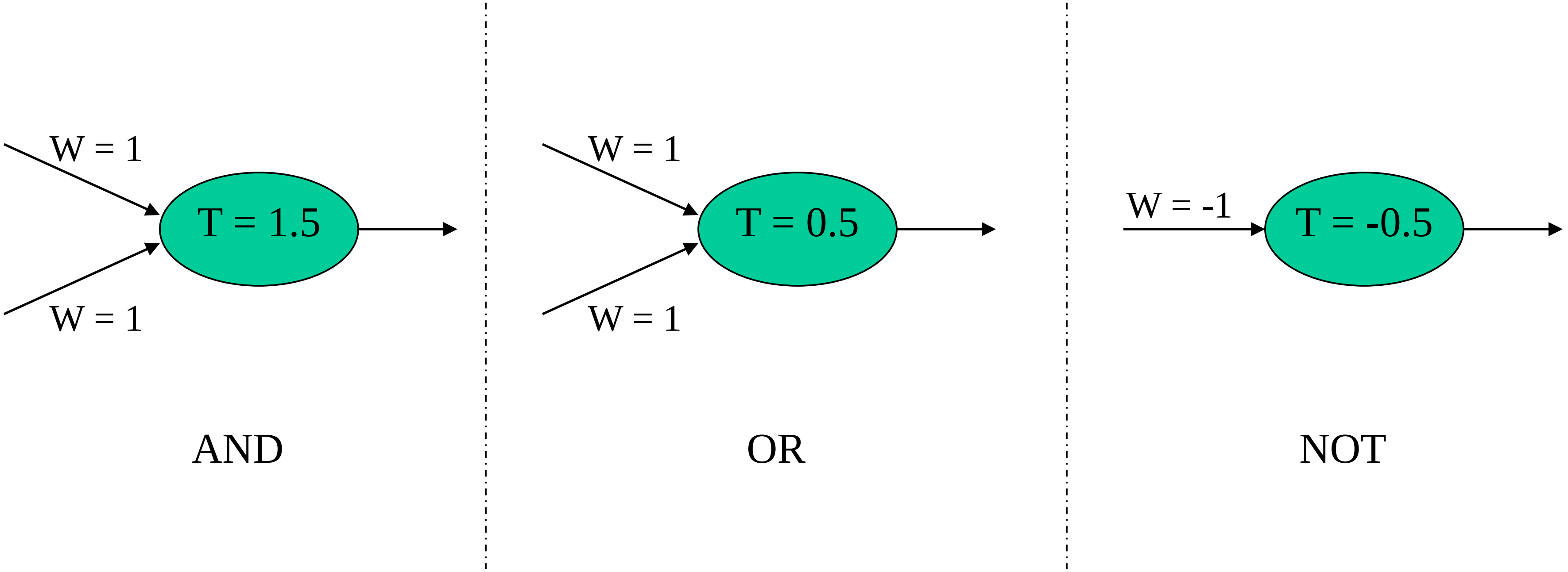
(a) Step function **(b) Sign function** **(c) Sigmoid function**

Three different activation functions for units.

$$\mathrm{step}_t(x) = \begin{cases} 1, & \text{if } x \geq t \\ 0, & \text{if } x < t \end{cases} \quad \mathrm{sign}(x) = \begin{cases} +1, & \text{if } x \geq 0 \\ -1, & \text{if } x < 0 \end{cases} \quad \mathrm{sigmoid}(x) = \frac{1}{1 + e^{-x}}$$

Non-linear functions!

Boolean Functions Using Step Activation Function



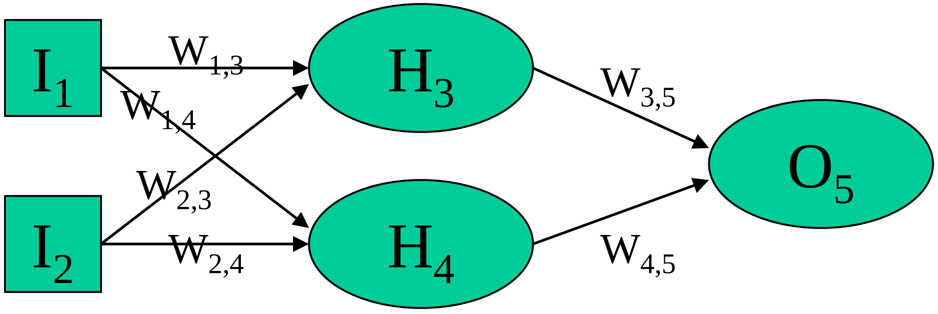
AND OR NOT

Network Structures

- Two main varieties
  - Feed-forward
    - Unidirectional links with no cycles
    - Directed acyclic graph (DAG)
  - Recurrent
    - Links can form arbitrary topologies
    - Contains cycles

Network Structures

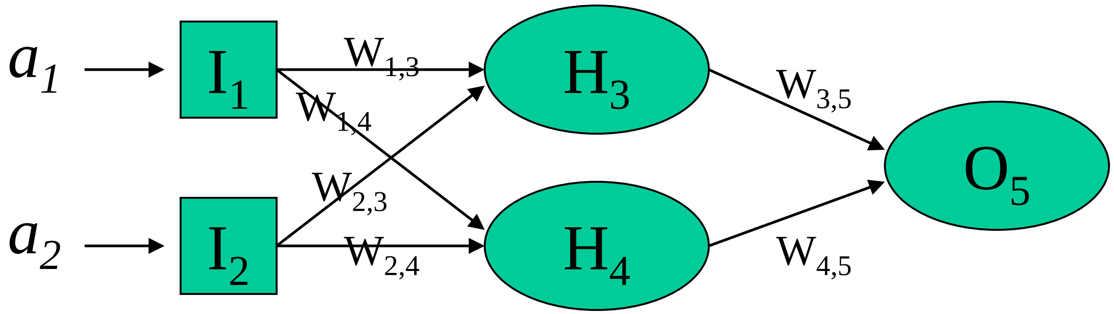
- Feed-forward networks have no internal state (other than weight values)
  - Simply computes function of input values using weights



Two-layer, feed-forward network

- Not like the brain!
  - We have memory
  - Many back connections

Network Structures



Two-layer, feed-forward network

Network calculates function (*g* is nonlinear activation):

$$O_5 = g(W_{3,5} \cdot g(W_{1,3}a_1 + W_{2,3}a_2) + W_{4,5} \cdot g(W_{1,4}a_1 + W_{2,4}a_2))$$

Learning just becomes a process of tuning parameters to fit data in training set!!!

Network Structures

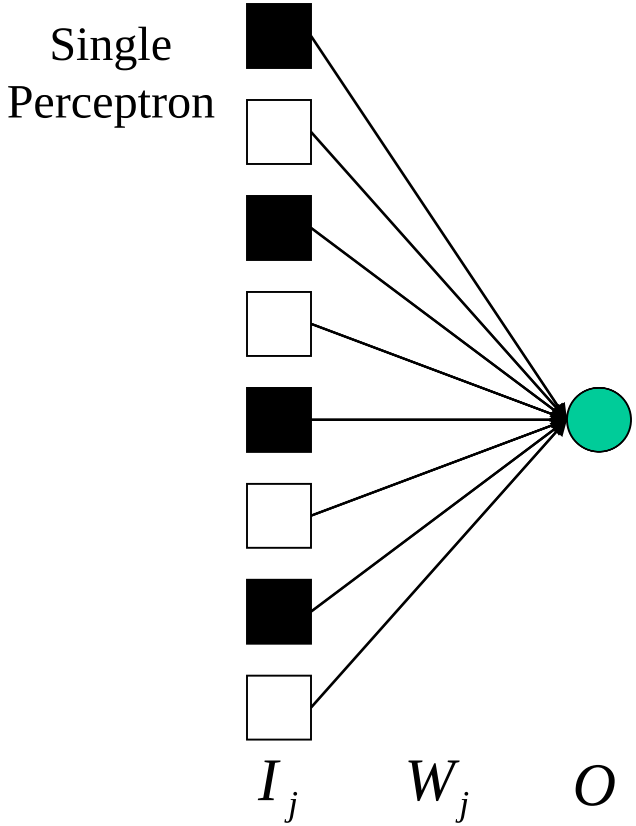
- Input units
  - Value of each unit determined by environment
- Output units
- Hidden units
  - Internal units that are neither input or output units
  - (Perceptrons have no hidden units)
- Multilayer networks
  - Networks with one or more layers of hidden units
  - One hidden layer
    - Theoretically can represent any continuous function of the inputs
  - Two hidden layers
    - Theoretically can represent even discontinuous functions

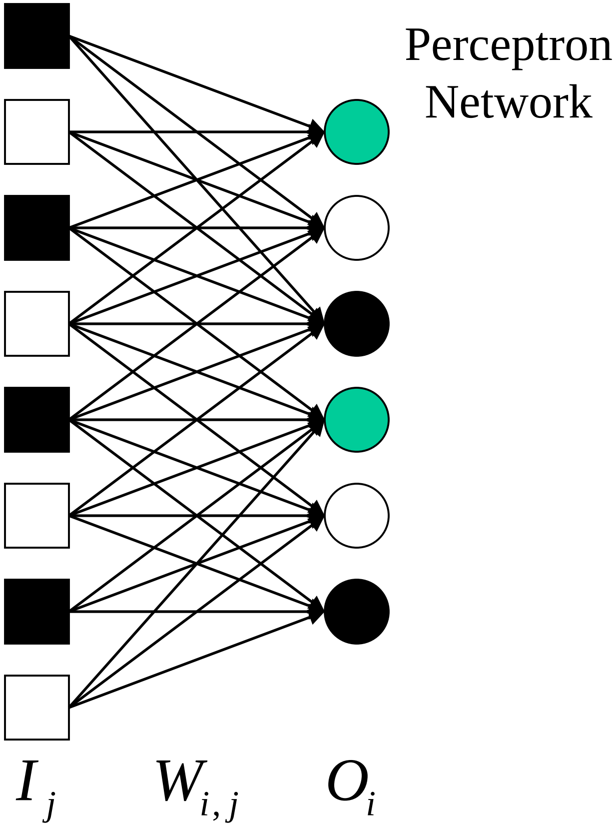
Optimal Network Structure

- Neural networks are subject to overfitting
  - When use too many parameters (weights) in model
  - Cross validation techniques are useful for determining right size of network

Perceptrons

- First studied in late 1950's
- Single-layer, feed-forward network





Perceptrons

- **Step activation** of output unit for (single) perceptron (I_0 = -1, W_0 = threshold)

$$O = \mathrm{Step}_0\left(\sum_j W_j\, I_j\right) = \mathrm{Step}_0(\boldsymbol{W} \cdot \boldsymbol{I})$$

- Perceptrons represent functions that are <u>linearly separable</u> ***

Dividing the Space (+,-)

- Consider, 2 inputs: x, y

$$O = \text{Step}_0(\boldsymbol{W} \cdot \boldsymbol{I}) = \text{Step}_0\left(\begin{bmatrix} 1 & -m & b \end{bmatrix} \cdot \begin{bmatrix} y \\ x \\ -1 \end{bmatrix}\right)$$
$$= \text{Step}_0(y - mx - b)$$

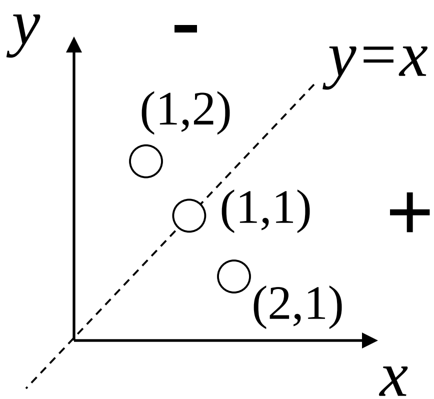
- The threshold of Step() is 0, so the important aspect: Is the input to step above or below 0?

? $\quad y - mx - b \geq 0$

$\quad\; y - mx - b < 0$ $\longrightarrow$ $y = mx + b$

Decision boundary

Dividing the Space (+,-)



$$0 = x - y$$

$$y = x$$

(1,2):

-1 = 1 - 2

(2,1):

+1 = 2 - 1

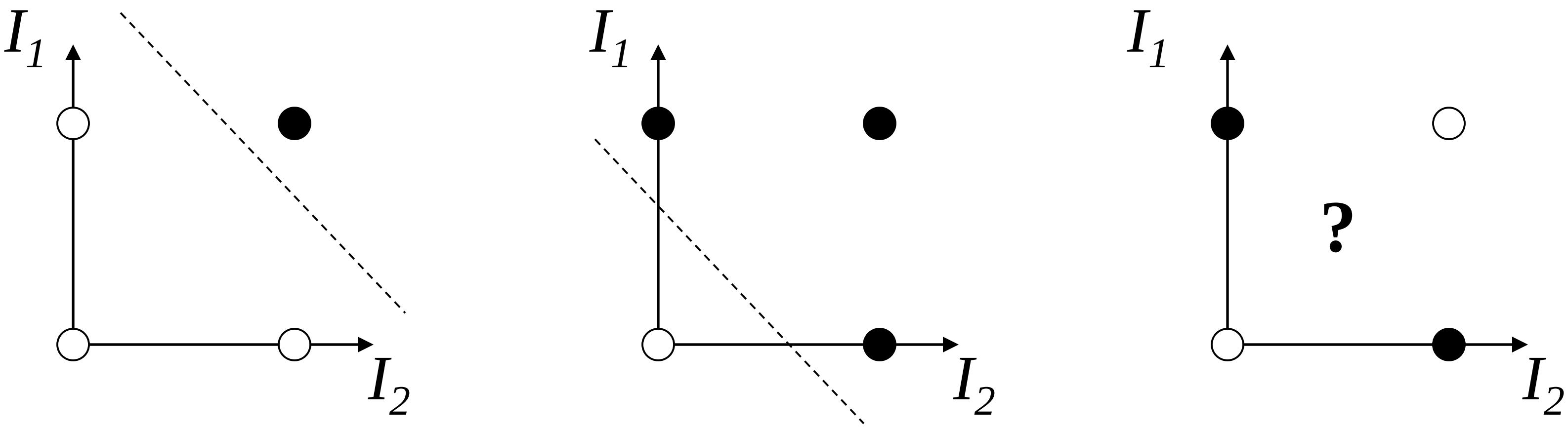
(1,1):

0 = 2 - 2

Linear Separability in Perceptrons

Limited in Boolean functions they can represent

AND, OR, but not XOR

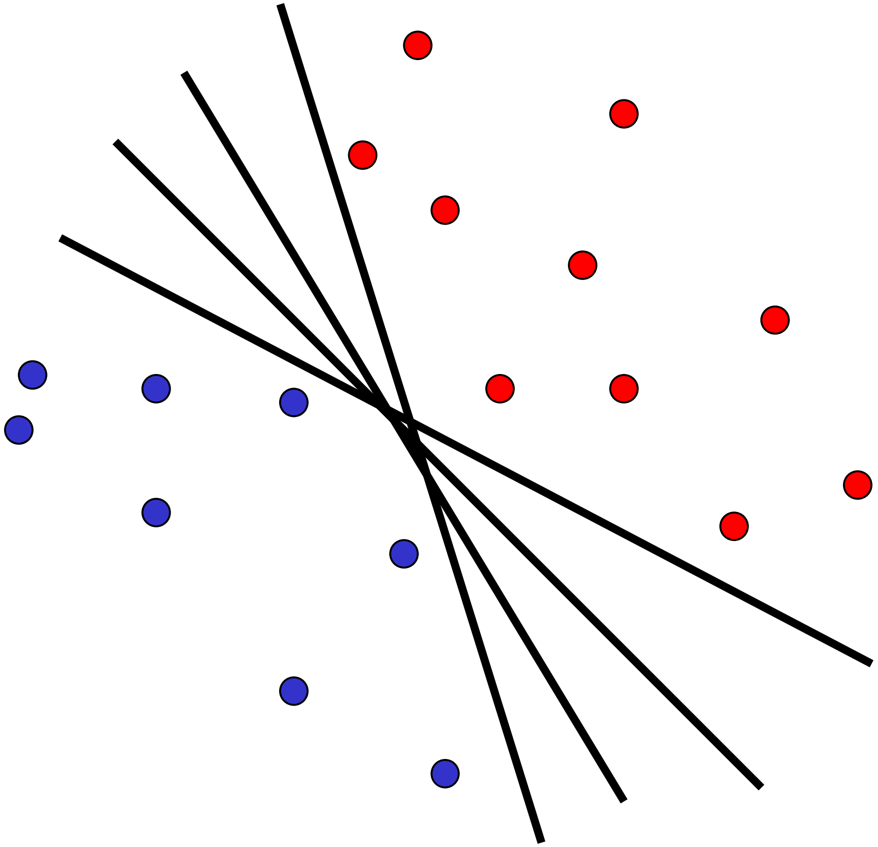


I_1 AND I_2 I_1 OR I_2 I_1 XOR I_2

"A perceptron can represent a function only if some line can separate all white dots from black dots"

Linear Classifiers

- Multiple Perceptron solutions to separate positive and negative examples



Perceptron Learning Algorithm

- Initially assign random weights [-0.5…0.5]
- Update network to try to make consistent with examples
  - Make small adjustments in weights to reduce difference between observed and predicted values
  - Updating process divided into "epochs"
    - Epoch involves updating all weights for all examples

Weight Updating via Gradient Descent

Error function:

$$E = \frac{1}{2} Err^2 = \frac{1}{2}\left[O - g\left(\sum_j W_j a_j \right) \right]^2$$

Desired output value

Perceptron output value

$$\begin{aligned} \frac{\partial E}{\partial W_j} &= \frac{1}{2}\frac{\partial Err^2}{\partial W_j} = Err \cdot \frac{\partial Err}{\partial W_j} \\ &= Err \cdot \frac{\partial}{\partial W_j}\left[O - g\left(\sum_j W_j a_j \right) \right] \\ &= Err \cdot g'() \cdot (-a_j) \end{aligned}$$

$$g' = sigmoid * (1 - sigmoid)$$

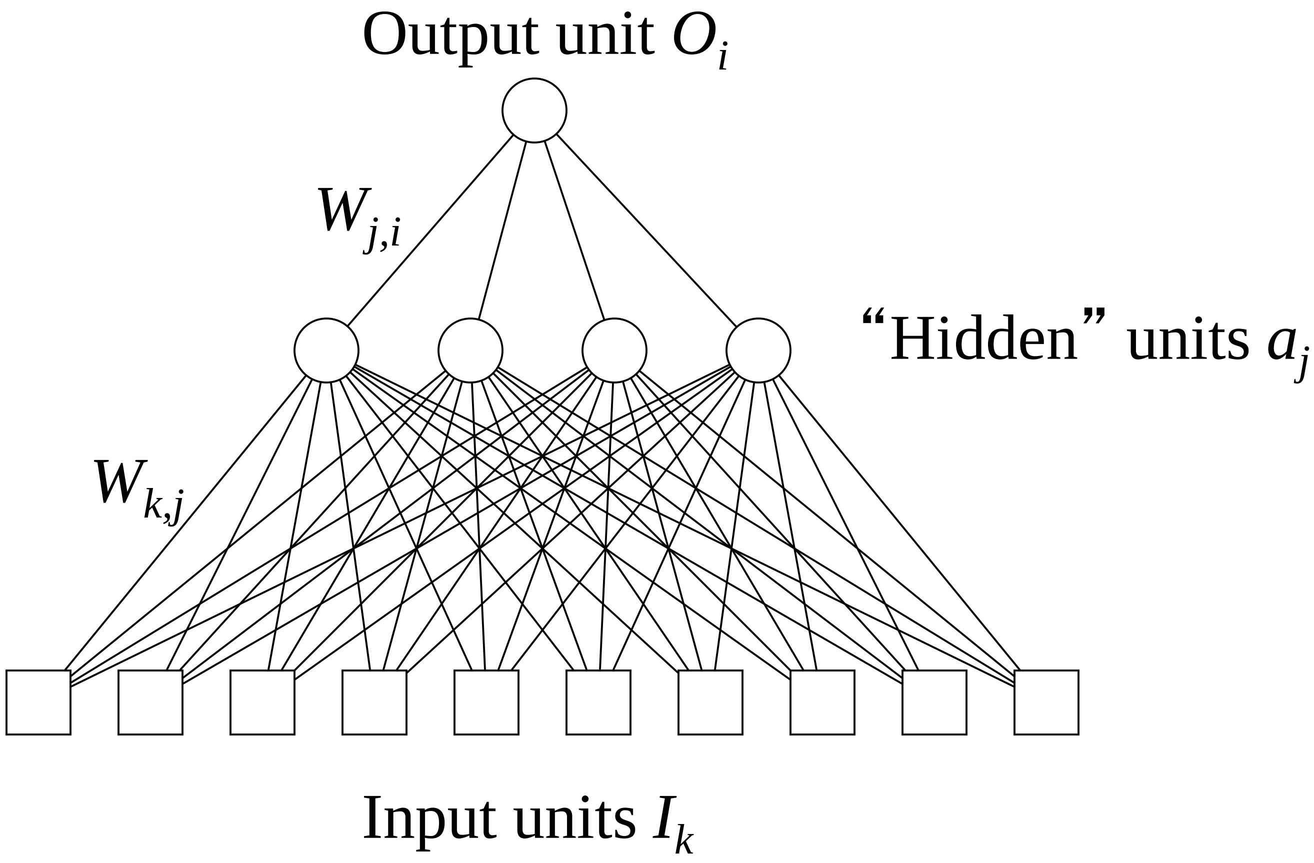
$$\begin{aligned} W_j &= W_j - \alpha \cdot \frac{\partial E}{\partial W_j} \\ &= W_j + \alpha \cdot Err \cdot g'() \cdot a_j \end{aligned}$$

Note that $g'()$ is omitted from "threshold" perceptrons

Perceptron Learning

- Perceptron convergence theorem is doing gradient descent through the weight space
- *Perceptrons*, by Minsky and Papert 1969
  - Clearly demonstrated the limits of linearly separable functions

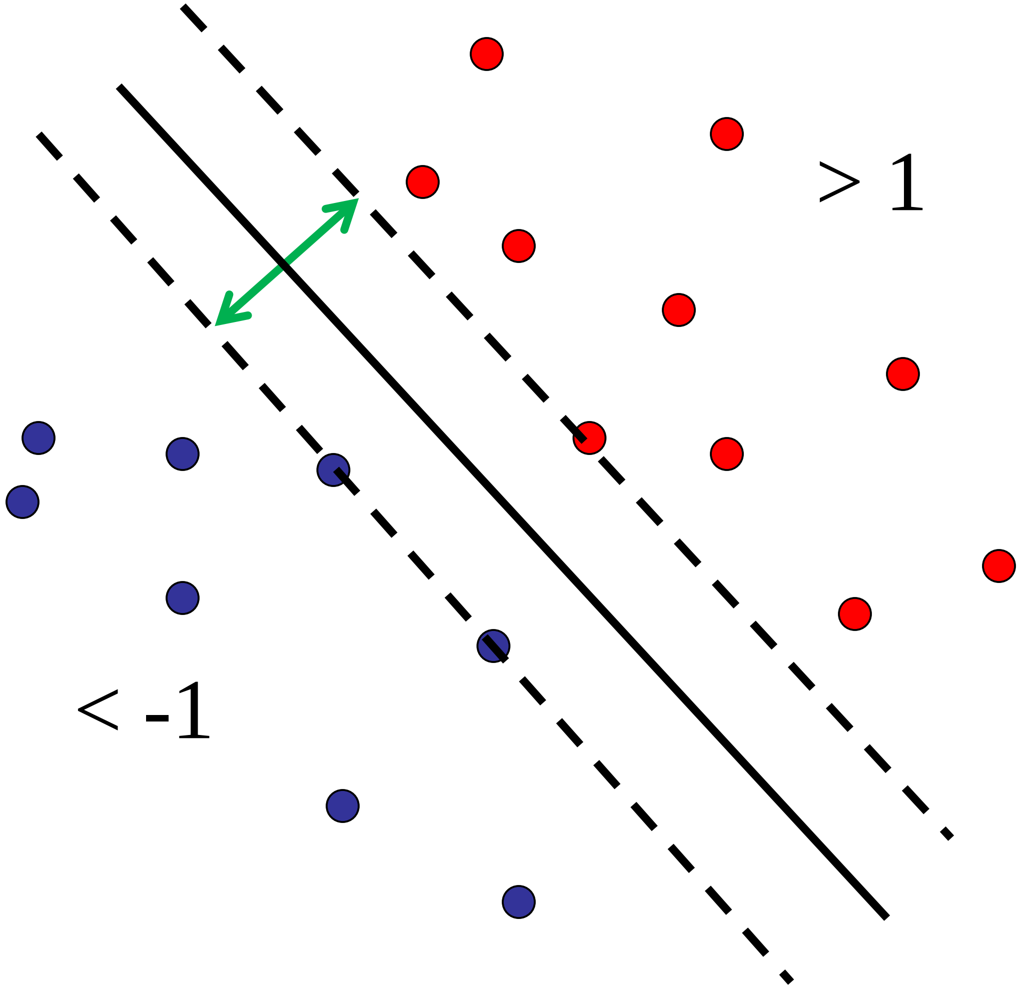
Multilayer Feed-Forward Networks



Learning in Multilayer Feed-Forward Networks

- Back-propagation learning algorithm
  - Assess blame for an error and divide it "locally" among contributing weights (divide contribution of each weight) and update layer by layer backwards

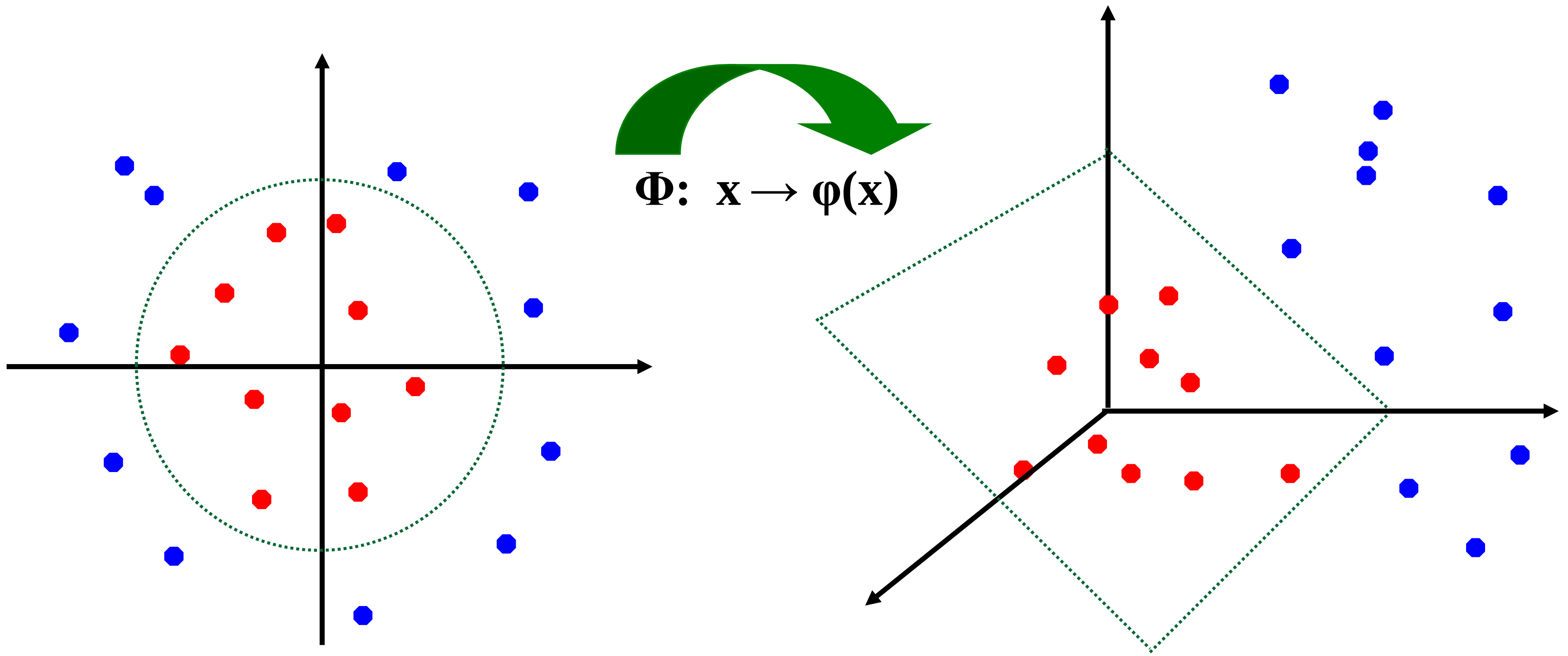
Support Vector Machines (SVMs)



- Discriminative classifier based on *optimal separating hyperplane (i.e., line for 2D case)*
- Maximize the ***margin*** between the positive and negative training examples

Non-Linear SVMs: Feature Spaces

- General idea: The original *input space* is mapped to some higher-dimensional *feature space* where the training set is more likely to be linearly separable:



Summary

- Neural net
  - Nodes, links, weights, activation level
- Each unit performs simple computation
  - Receives signals from input links
  - Computes new activation level
  - Sends activation level along each output link
- Feed-forward network
  - Unidirectional links with no cycles
- Perceptrons
  - Single-layer, feed-forward network
  - Represent functions that are linearly separable
- Back-propagation for multilayer networks